Magnetic monopoles in a charged two-condensate Bose-Einstein system

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We propose that a charged two-condensate Bose system possesses point-like topological defects which can be interpreted as magnetic monopoles. By making use of the ϕ-mapping theory, the topological charges of these magnetic monopoles can be expressed in terms of the Hopf indices and Brouwer degree of the ϕ-mapping.

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Superconductivity in magnesium diboride (MgB_2) with a remarkably high $T_c = 39$ K was recently reported by Nagamatsu *et al.*[1] Since then, great attention has been directed towards understanding the detailed nature of superconductivity in this simple intermetallic compound. Because the superconductivity of MgB_2 possesses an isotope effect consistent with the phonon-mediated electron pairing of the BCS theory, but with an extremely high critical temperature, it has reopened the question of the maximum T_c that can be produced by that mechanism .[2,3,4,5] Furthermore, it raises a new question: what is the mechanism of the superconductivity of MbB_2, and, in particular, whether this is a one- or two-gap superconductor.

The evidence for multiple gap structure involves tunneling measurements of the gap. Values of $2\Delta/k_BT_c$ ranging from 1.2 to 4 have been reported[6]. The values below the BCS weak-coupling limit of 3.5 have been attributed to surface effects, but the best-quality spectra[7] show a very clean gap with the number equal to 1.25. Sharvin contact measurements[8] reveal a gap at 4.3 meV ($2\Delta/k_BT_c = 2.6$), and additional structures at $2\Delta/k_BT_c = 1.5$ and 3, raising the possibility of multiple gaps. Careful analysis of the temperature and magnetic-field dependence of the specific heat suggests multiple gap structure as well[2,3]. In fact, there exist two gaps of different magnitude associated with different bands in MgB_2. Their ratio is estimated as $r = \Delta_0^S/\Delta_0^L \sim 0.3 - 0.4$[9] where the larger gap Δ_0^L is associated with the two-dimensional σ-bands and the smaller gap Δ_0^S with the three-dimensional π-bands[10].

Moreover, the experiment of scanning tunneling spectroscopy (STS) measurements on single crystal MgB_2 shows that the coherent length in the π band is approximately 50 nm[11] which is much larger than an estimate which one would obtain from a standard GL formula. All these spark renewed interest to two-gap superconductivity.

Two-gap superconductivity is now supported by an increasing number of experimental reports[12]. Two-gap or two-band superconductivity was first studied in the 1950s[13] and has now found renewed relevance in MgB_2. In addition, and contrary to many materials or alloys studied earlier, the two bands in MgB_2 have a roughly equal filling factor, opening the possibility for interesting new phenomena.

Principly, the two-gap superconductivity can be investigated in the frame of the charged two-condensate Bose system. This system is described by a Ginzburg-Landau model with two flavors of Cooper pairs. Alternatively, it relates to a Gross-Pitaevskii functional with two charged condensates of tightly bound fermion pairs, or some other charged bosonic fields. Such theoretical models have a wide range of applications and have been previously considered in connection with two-band superconductivity[14,15,16,17]. By making use of this theoretical model, vortices with fractional flux in two-gap superconductors has been presented[16].

The works mentioned above stimulate us to investigate the topological structure of the charged two-condensate Bose-Einstein system in more detail. Let us consider a Bose-Einstein system with two electromagnetically coupled, oppositely charged condensates, this system can be described by a two-flavor Ginzburg-Landau-Gross-Pitaevkii (GLGP) functional[18],

$$F = \frac{1}{2m_1}\left|\left(\hbar\partial_\mu + i\frac{2e}{c}A_\mu\right)\Psi_1\right|^2 + \frac{1}{2m_2}\left|\left(\hbar\partial_\mu - i\frac{2e}{c}A_\mu\right)\Psi_2\right|^2 + V(\Psi_{1,2}) + \frac{\mathbf{B}^2}{8\pi}, \quad (1)$$

in which

$$V(\Psi_{1,2}) = -b_\alpha|\Psi_\alpha|^2 + \frac{c_\alpha}{2}|\Psi_\alpha|^4 + \eta[\Psi_1^*\Psi_2 + \Psi_2^*\Psi_1], \quad (2)$$

where η is a characteristic of interband Josephson coupling strength[19]. What is much more important in the present GLGP model is that the two charged fields are not independent but nontrivially coupled through the electromagnetic field and the Josephson coupling. This kind of nontrivial coupling indicates that in this system there should be a nontrivial, hidden topology which, however, cannot be recognized obviously in the form of Eq. (1). In order to find out the topological structure and to investigate it conveniently, we need to reform the GLGP functional.

By introducing a new set of variables ρ and $\chi_{1,2}$ by

$$\Psi_\alpha = \sqrt{2m_\alpha}\rho\chi_\alpha, \quad (3)$$

where the complex $\chi_\alpha = |\chi_\alpha|e^{i\varphi_\alpha}$ satisfying $|\chi_1|^2 + |\chi_2|^2 = 1$ and the modulus taking the form

$$\rho^2 = \frac{1}{2}\left(\frac{|\Psi_1|^2}{m_1} + \frac{|\Psi_2|^2}{m_2}\right), \quad (4)$$

the original GLGP free energy density (1) can be represented as

$$F = \frac{\hbar^2\rho^2}{4}(\partial\vec{n})^2 + \hbar^2(\partial\rho)^2 + \frac{\hbar^2 c^2}{512\pi e^2}\Bigg(\frac{1}{\hbar}[\partial_\mu C_\nu - \partial_\nu C_\mu] - \vec{n}\cdot\partial_\mu\vec{n}\times\partial_\nu\vec{n}\Bigg)^2 + \frac{\rho^2}{16}\vec{C}^2 + V(\rho, n_1, n_3), \tag{5}$$

where

$$C_\mu = 2i\hbar[\chi_1\partial_\mu\chi_1^* - \chi_1^*\partial_\mu\chi_1 - \chi_2\partial_\mu\chi_2^* + \chi_2^*\partial_\mu\chi_2] - \frac{8e}{c}A_\mu \tag{6}$$

$$\vec{n} = (\bar{\chi}, \vec{\sigma}\chi), \tag{7}$$

in which $(\quad , \quad)$ denotes the scalar product and $\bar{\chi} = (\chi_1^* \quad \chi_2^*)$, $\vec{\sigma}$ stand for the Pauli matrices. Now we find that there exists an exact equivalence between the two-flavor GLGP model and the nonlinear O(3) σ model[20] which is much more important to describe the topological structure in high energy physics. By analogy with it, the counterpart in condensed matter has been discussed which showed that there are topological excitations in the form of stable, finite length knotted closed vortices in two-condensate Bose system[18]. In this paper, we will show that, beside the knotted vortices, another kind of topological defects, namely the magnetic monopoles, also possibly exist in this system. This kind of topological defects, magnetic monopoles, has also been discussed in chiral supercondoctors and superfluids[21].

As shown in Eq.(5), we know that the magnetic field of the system can be divided into two parts, one is the contribution of field C_μ, from Eq.(6) we learn that this part is introduced by the supercurrent density[18] and can only present us with the topological defects named vortices, as what in the single-condensate system. Another part, the contribution $\vec{n}\cdot\partial_\mu\vec{n}\times\partial_\nu\vec{n}$ to the magnetic field term in Eq.(5), is a fundamentally important property of the two-condensate system which has no counterpart in a single condensate system. Indeed, it is exactly due to the presence of this term that the two-condensate system acquires properties which are qualitatively very different from those of a single-condensate system: This term describes the magnetic field that becomes induced in the system due to a nontrivial electromagnetic interaction between the two condensates. Thus it is needed to investigate this induced magnetic field in detail.

The induced magnetic field $\tilde{B}_\mu$ is expressed as

$$\tilde{B}_\mu = \frac{\hbar c}{8e}\epsilon_{\mu\nu\lambda}\epsilon_{abc}n^a\partial_\nu n^b\partial_\lambda n^c. \tag{8}$$

Let us now investigate the divergence of the induced magnetic field $\tilde{B}_\mu$, namely Q, which can be represented in terms of the unit vector field n^a as

$$Q = \partial_\mu\tilde{B}_\mu = \frac{\hbar c}{8e}\epsilon_{\mu\nu\lambda}\epsilon_{abc}\partial_\mu n^a\partial_\nu n^b\partial_\lambda n^c. \tag{9}$$

Usually, a unit vector can be written as $\vec{n} = (\sin\theta\cos\gamma, \sin\theta\sin\gamma, \cos\theta)$, which was adopted in many previous works. With this normal expression, one can immediately draw the conclusion that the divergence of the induced magnetic field is equal to zero, as the Maxwellian equation shows.

In fact, in the present context, the unit vector is defined by Eq. (7), from which it is very easy to get $||\vec{n}||^2 = (|\chi_1|^2 + |\chi_2|^2)^2$, and the components of the unit vector $\vec{n}$ are determined by χ_α. The condition $|\chi_1|^2 + |\chi_2|^2 = 1$ we mentioned below Eq. (3) safeguards that the vector $\vec{n}$ is a unit vector in the whole space and there is no point at which this condition can be destroyed. Now let us consider about the components of field χ_α, as well as the components of the unit vector n^a. Normally, the components χ_α are well defined by Eq. (3), so are the components n^a. In this case, the unit vector field n^a can be reduced to the normal one and the result of the divergence Q is equal to zero, just as mentioned above. However, when we investigate the behavior of the original system described in Eq. (1) at the points where both Ψ_1 and Ψ_2 are zero, the situation is expected to be changed. After reforming the GLGP functional (1) via ρ and $\chi_{1,2}$, we find that even though the condition $|\chi_1|^2 + |\chi_2|^2 = 1$ is still held, the components χ_α are not well defined at these points, this is clear from Eqs.(3) and (4). The same situation happens to $\vec{n}$, at these points, the unit vector condition of $\vec{n}$ is still valid, however the components of the unit vector n^a are undetermined. This indetermination indicates that these zero points are the topological singular points of the system, and it leads to an unusual behavior of $\tilde{B}_\mu$ and Q at these points. Hence, in order to investigate the unusual property of Q at these special points, we represent the unit vector n^a as

$$n^a = \frac{\phi^a}{||\phi||}, \quad ||\phi|| = \sqrt{\phi^a\phi^a}. \tag{10}$$

As discussed above, this is a reasonable representation, ϕ^a is a three component vector field related to the order parameters Ψ_α via Eqs.(7) and (3). Obviously, it can be looked upon as a smooth mapping between the three-dimensional space X (with the local coordinate x) and the three-dimensional Euclidean space R^3 $\phi : x \mapsto \vec{\phi}(x) \in R^3$, and n^a a section of the sphere bundle $S(X)$. Clearly, the zero points of $\phi^a(x)$ correspond to the points where Ψ_1 and Ψ_2 are zero, We would like to name these points as the singular points of n^a. In the following, by virtue of the so called ϕ-mapping method[22,23], we will show that, when ϕ^a field possesses several zero points, the divergence of the induced magnetic field is no longer zero but takes the form of the δ-function.

From (10), we have

$$\partial_\mu n^a = \frac{1}{||\phi||}\partial_\mu\phi^a + \phi^a\partial_\mu\left(\frac{1}{||\phi||}\right). \tag{11}$$

Due to these expressions, the divergence Q can be rewrit-

ten as

$$Q=-\frac{\hbar c}{8e}\epsilon_{\mu\nu\lambda}\epsilon_{abc}\partial_\mu\phi^d\partial_\nu\phi^b\partial_\lambda\phi^c\frac{\partial}{\partial\phi^d}\frac{\partial}{\partial\phi^a}\left(\frac{1}{||\phi||}\right) \quad (12)$$

If we define the Jacobian $D(\phi/x)$ as

$$\epsilon^{abc}D(\phi/x)=\epsilon_{\mu\nu\lambda}\partial_\mu\phi^a\partial_\nu\phi^b\partial_\lambda\phi^c \quad (13)$$

and make use of the Laplacian Green function relation in ϕ-space,

$$\frac{\partial}{\partial\phi^d}\frac{\partial}{\partial\phi^a}\left(\frac{1}{||\phi||}\right)=-4\pi\delta(\vec{\phi}), \quad (14)$$

we do obtain the δ-function structure of the divergence of the induced magnetic field as is expected,

$$Q=\partial_\mu\tilde{B_\mu}=\frac{\hbar\pi c}{2e}\delta(\vec{\phi})D(\phi/x), \quad (15)$$

which shows that the divergence of the induced magnetic field $\tilde{B_\mu}$ is not equal to zero. This indicates that in the two-condensate Bose system, there are point-like topological defects (or namely, magnetic monopoles) located at the zero point of field $\vec{\phi}$.

Then questions are raised naturally: what is the topological charges of the magnetic monopoles and how to obtain the inner structure of Q.

From the above, we see that the zero point of $\vec{\phi}$ plays an important role. We discuss them more deeply. Suppose the function $\phi^a(x)$ possesses l isolated zeroes. The implicit function theorem expresses that when these zeroes are regular points of ϕ-mapping and require the Jacobian $D(\phi/x)\neq 0$, the zero points can be expressed by

$$\vec{x}=\vec{z}_i, \quad i=1,...,l. \quad (16)$$

Now, we will investigate the topological charges of the magnetic monopoles and their quantization. Let M_i be a neighborhood of $\vec{z}_i$ with boundary ∂M_i satisfying $\vec{z}_i\notin\partial M_i$, $M_i\cap M_j=\emptyset$. Then the generalized winding number W_i of $n^a(\vec{x})$ at $\vec{z}_i$ can be defined by the Gauss map[24,25] n: $\partial M_i\rightarrow S^2$,

$$W_i=\frac{1}{8\pi}\int_{\partial M_i}n^*(\epsilon_{abc}n^a dn^b\wedge dn^c), \quad (17)$$

where n^* is the pull back of map n. The generalized winding number is a topological invariant and is also called the degree of Gauss map. It is well known that W_i are corresponding to the second homotopy group $\pi_2[S^2]=Z$ (the set of integers). Using the Stokes' theorem in exterior differential form and the result in (15), we get

$$W_i=\int_{M_i}\delta(\vec{\phi})D(\phi/x)d^3x. \quad (18)$$

By analogy with the procedure of deducing $\delta(f(x))$, one can expand the δ-function $\delta(\vec{\phi})$ as

$$\delta(\vec{\phi})=\sum_{i=1}^{l}c_i\delta(\vec{x}-\vec{z}_i), \quad (19)$$

where the coefficients c_i must be positive, i.e. $c_i=|c_i|$. Substituting (19) into (18) and calculating the integral, we get an expression for c_i,

$$c_i=\frac{|W_i|}{|D(\phi/x)_{\vec{x}=\vec{z}_i}|}. \quad (20)$$

Letting $|W_i|=\beta_i$, using this expansion of $\delta(\vec{\phi})$, it is evident that Q in (15) can be further expressed in the form

$$Q=\frac{\hbar\pi c}{2e}\sum_{i=1}^{l}\beta_i\eta_i\delta(\vec{x}-\vec{z}_i), \quad (21)$$

where the positive integer β_i is the so-called Hopf index of the ϕ-mapping on M_i, $\eta_i=\mathrm{sign}D(\phi/x)_{\vec{z}_i}=\pm 1$ is the Brouwer degrees of the ϕ-mapping. This expression is exactly the density of the system of l topological defects (magnetic monopoles) with corresponding topological charges $g_i=\beta_i\eta_i$, the Hopf indices β_i characterize the absolute value of the monopole topological charges and the Brouwer degrees $\eta_i=1$ correspond to monopoles while $\eta_i=-1$ to antimonopoles.

In summary, by making use of the ϕ-mapping method, we find that, beside the closed knotted vortices[18], there is another kind of topological defects, namely the magnetic monopoles, in two-band superconductors. Moreover, these monopoles are located at the zero points of the field $\vec{\phi}$, or namely, at the singular points of the field $\vec{n}$ defined in Eq.(7), and their topological charges can be expressed in terms of the Hopf indices and the Brouwer degrees of the ϕ–mapping. This is very similar with a different physical system discussed by Cho[26].

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1 J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani and J. Akimitsu, Nature (London) **410**, 63 (2001)

2 Y. Wang, T. Plackowski and A. Junod, Physica C **355C**, 179 (2001)

3 F. Bouquet, R. A. Fisher, N. E. Phillips, D. G. Hinks and J. D. Jorgensen, Phys. Rev. Lett. **87**, 047001 (2001)

4 J. Kortus, I. I. Mazin, K. D. Belashchenko, V. P. Antropov

and L. L. Boyer, Phys. Rev. Lett. **86**, 4656 (2001)
[5] P. Canfield, D. K. Finnemore, S. L. Bud'ko, J. E. Ostenson, G. Lapertot, C. E. Cunningham and C. Petrovic, Phys. Rev. Lett. **86**, 2423 (2001)
[6] A. Y. Liu, I. I. Mazin and Jens Kortus, Phys. Rev. Lett **87**, 087005 (2001)
[7] G. Rubio-Bollinger, H. Suderow, and S. Vieira, Phys. Rev. Lett. **86**, 5582 (2001)
[8] H. Schmidt, J. F. Zasadzinski, K. E. Gray and D. G. Hinks, Phys. Rev. B **63**, 220504 (2001)
[9] P. Szabó *et al*, Phys. Rev. Lett. **87**, 137005 (2001)
[10] L. Tewordt and D. Fay, cond-mat/0212503 (2002)
[11] M. R. Eskildsen *et al*, Phys. Rev. Lett. **89**, 187003 (2002)
[12] X.K. Chen *et al.*, Phys. Rev. Lett. **87**, 157002 (2001); F. Giubileo *et al.*, *ibid.* **87**, 177008 (2001); F. Bouquet *et al.*, Europhys. Lett. **56**, 856 (2001); H. Schmidt *et al.*, Phys. Rev. Lett. **88**, 127002 (2002); A. Junod *et al.*, in *Studies of High Temperature Superconductors*, edited by A. Narlikar (Nova Publishers, Commack, NY, 2002), Vol. 38, p. 179; M. Iavarone *et al.*, Phys. Rev. Lett. **89**, 187002 (2002)
[13] For a review, see G. Gladstone, M.A. Jensen, and J.R. Schrieffer, in *Superconductivity*, edited by R.D. Parks, Vol.2, p. 665 (Marcel Dekker, New York, 1969)
[14] H. Suhl, B. T. Matthias, and L. R. Walker, Phys. Rev. Lett. **3**, 552 (1959)
[15] J. Kondo, Prog. Theor. Phys. **29**, 1 (1963); D. R. Tilley, Proc. Phys. Soc. **84**, 573 (1964); A. J. Leggett, Prog. Theor. Phys. **36**, 901 (1966)
[16] E. Babaev, Phys. Rev. Lett. **89**, 067001 (2002)
[17] E. Babaev, Phys. Rev. Lett. **88**, 177002 (2002); cond-mat/0302218 (2003)
[18] E. Babaev, L. D. Faddeev, and A. J. Niemi, Phys. Rev. B **65**, 100512 (2002)
[19] A. J. Leggett, Prog. Theor. Phys. **36**, 901 (1966); E. Babaev, cond-mat/0201547
[20] L. Faddeev, in *Relative, Quanta and Cosmology* Vol.1, p. 247, edited by M. Pantaleo and F. De Finis (Johnson Reprint, New York, 1979); L. Faddeev and A.J. Niemi, Nature (London) **387**, 58 (1997); Phys. Rev. Lett. **82**, 1624 (1999)
[21] G. E. Volovik, Proc. Nat. Acad. Sci. **97**, 2431 (2000); Pisma Zh. Eksp. Teor. Fiz. **70**, 776 (1999); JETP Lett. **70**, 792 (1999)
[22] Y. Jiang and Y. S. Duan, J. Math. Phys. **41**, 6463 (2000)
[23] Y. Jiang and Y. S. Duan, J. Math. Phys. **41**, 2616 (2000)
[24] Y. S. Duan, Y. Jiang and T. Xu, Phys. Lett. A **252**, 307 (1999)
[25] B. A. Dubrovin, A. T. Fomenko and S. P. Novikov,*Modern geometry method and application*, Part II (Springer Verlag, New York 1975); V. Guillemin and A. Pollack, *Differential Topology* (Prentice Hall, Englewood Cliffs, 1974); J. W. Milnor, *Topology, from the differential viewpoint* (University Press of Virginia, Charlottesville, 1965)
[26] Y. M. Cho, Phys. Rev. Lett. **87**, 252001 (2001)